THE STATUS OF THE PROBLEMS FROM THE BOOK

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1. Introduction

In this article we report on the status of the open problems in *The Structure of Finite Algebras*, by David Hobby and Ralph McKenzie, [3]. Any article of this type surely must include an account of what is known concerning the seventeen problems listed at the end of [3]. It is conceivable that a thorough article of this type might even include comments on implied problems, improvements to theorems, and extensions of the basic theory. We restrict the scope of this article to problems in the text or on the problem list that Hobby and McKenzie specifically identify as open problems.

We located four open problems from the text (labelled Problems *A*–*D*), which we include with the seventeen on the Problem List (labelled Problems 1–17). Below we reproduce the problem, describe its status, and include notes and relevant citations to the literature.

Problem A. (See Exercise 8.8 (1).) Is it true that $\mathcal{V}$ is congruence permutable if and only if

(i) $\text{typ}\{\mathcal{V}\} \subseteq \{\mathbf{2}, \mathbf{3}\}$; and
(ii) For every finite $\mathbf{A} \in \mathcal{V}$ and $\alpha \prec \beta$ in $\text{Con}(\mathbf{A})$ and $\langle a, b\rangle \in \beta - \alpha$, there exists $u \equiv b \pmod{\alpha}$ with $\{a, u\} \subseteq N$ for some $\langle \alpha, \beta\rangle$-trace N?

Status. Solved affirmatively.

Notes. Theorem 9.14 of [3] proves that a variety is congruence n-permutable for some n if and only if condition (i) holds. Exercise 8.8 (1) asks the reader to prove that if a variety is congruence $(2-)$permutable, then (i) and (ii) hold. The exercise then raises as an open question whether (i) and (ii) actually characterize congruence permutability. The first proof that these conditions do characterize congruence permutability was discovered by M. Valeriote and R. Willard (see [26]). K. A. Kearnes later provided another proof in [4], and gave a similar characterization of congruence 3-permutability.

Problem B. (See page 143.) Does congruence SD($\vee$) for the finite algebras in the locally finite variety $\mathcal{V}$ imply congruence SD($\vee$) for all algebras in $\mathcal{V}$?

Status. Solved affirmatively.

Notes. This question is motivated by Theorem 9.11 of [3], which proves that a locally finite variety omits types $\mathbf{1}, \mathbf{2}$ and $\mathbf{5}$ if and only if the finite algebras in

$\mathcal{V}$ are congruence SD($\vee$). The afirmative answer to this question was proved by K. A. Kearnes in [7].

Problem C. (See Exercise 14.9 (2).) Let $\mathcal{V} = \mathcal{V}(\mathbf{B}_1, \dots, \mathbf{B}_n)$ where the $\mathbf{B}_i$ are finite. Assume that $\mathcal{V}$ omits types $\mathbf{1}$ and $\mathbf{5}$ and that k is the supremum of $\#(\gamma/\delta)$ with $\langle \delta, \gamma \rangle$ ranging over prime quotients of the $\mathbf{B}_i$. Is it true that $\#(\beta/\alpha) \leq k$ when $\mathbf{A} \in \mathcal{V}$ and $\alpha \prec \beta$ in $\mathrm{Con}(\mathbf{A})$?

Status. Solved negatively.

Notes. Exercise 14.9 (2) asks the reader to prove that the statement is true if $\mathcal{V}$ is congruence modular, and asks whether congruence modularity can be weakened to $\mathrm{typ}\{\mathcal{V}\} \cap \{\mathbf{1}, \mathbf{5}\} = \emptyset$. A counterexample can be found in [8] by K. A. Kearnes, E. W. Kiss, Á. Szendrei, and R. Willard.

Problem D. (See Exercise 14.9 (3).) Do there exist nonisomorphic simple algebras of type $\mathbf{2}$ that generate the same variety?

Status. Solved negatively.

Notes. Theorem 14.8 of [3] proves that if two finite simple algebras generate the same variety, then they must have the same type. Moreover, it is shown that if the type is $\mathbf{3}$ or $\mathbf{4}$, then the two simple algebras are isomorphic. On the other hand, Exercise 14.9 (3) shows that there exist nonisomorphic simple algebras of type $\mathbf{1}$ that generate the same variety, and Exercise 14.9 (4) does the same for type $\mathbf{5}$. The type $\mathbf{2}$ case is the only case left open in [3].

It is proved in [11] by K. A. Kearnes and Á. Szendrei that if two type $\mathbf{2}$ simple algebras generate the same variety, then they are isomorphic.

Problem 1. Is it true that if $\mathbf{A}$ is an Abelian algebra and there is an equation in the language of lattices that is valid in the congruence lattice of every algebra in $\mathcal{V}(\mathbf{A})$, and fails to be valid in some lattice, then $\mathbf{A}$ must be polynomially equivalent to a module?

Status. Solved affirmatively.

Notes. One can find an affirmative solution for locally finite varieties in Exercise 9.20 (2). The affirmative solution for nonlocally finite varieties is due to K. A. Kearnes and Á. Szendrei (see [12]). They show that if $\mathbf{A}$ is an Abelian algebra in a variety that satisfies a nontrivial idempotent Maltsev condition, then $\mathbf{A}$ is quasi-affine, and that if the Maltsev condition fails in the variety of semilattices then it is affine.

Problem 2. A variety $\mathcal{V}$ is called Abelian iff all its algebras are Abelian, and Hamiltonian iff every subalgebra of an algebra $\mathbf{A}$ in $\mathcal{V}$ is a full equivalence class of a congruence on $\mathbf{A}$. Is every locally finite Abelian variety Hamiltonian?

Status. Solved affirmatively.

Notes. This problem was solved by E. W. Kiss and M. A. Valeriote in [18]. The result is considered to be important in the study of locally finite Abelian varieties, because it is known that any locally finite Hamiltonian variety has Klukovits terms (see [19]), the congruence extension property (see [16]), and definable principal congruences (see [1]).

In an earlier paper, [17], Kiss and Valeriote introduced the strong Hamiltonian property and proved that a locally finite variety is strongly Abelian iff it is strongly Hamiltonian. They also provided an example to show that a nonlocally finite strongly Abelian variety may fail to be Hamiltonian.

Problem 3. If $\mathbf{A}$ is a finite Abelian algebra of finite type, is $\mathcal{V}(\mathbf{A})$ finitely axiomatizable?

Status. Solved negatively.

Notes. In [15], K. A. Kearnes and R. Willard exhibit a 6-element nonfinitely axiomatizable Abelian algebra. In fact, the algebra they exhibit even generates an Abelian variety. By contrast, in [14] Kearnes and Willard prove that a locally finite Abelian variety of finite type cannot be inherently nonfinitely based.

Problem 4. Let $\mathbf{A}$ be a finite algebra with congruence lattice $\mathbf{L}$. Suppose that $\mathbf{L}$ has an $\mathbf{M}_n$ ($n \geq 3$) as a sublattice in which the 0 and 1 are the same as in $\mathbf{L}$. Is $\mathbf{A}$ Abelian?

Status. Solved negatively.

Notes. It follows from Corollary 5.8 of [3] that if $\mathbf{L}$ equals $\mathbf{M}_n$ ($n \geq 3$), then $\mathbf{A}$ is abelian. It follows from Theorem 7.7 of [3] that if $\mathbf{L}$ has an $\mathbf{M}_n$ ($n \geq 3$) as a $0,1$–sublattice, then $\mathbf{A}$ is at least solvable; moreover, by results in [5], if $\mathbf{1} \notin \operatorname{typ}\{\mathbf{A}\}$ then $\mathbf{A}$ is even Abelian. Although these results suggest an affirmative answer to Problem 4, R. Willard found a way to construct nonabelian finite algebras $\mathbf{A}$ whose congruence lattice $\mathbf{L}$ has $\mathbf{M}_n$ as a $0,1$–sublattice. Any such algebra must be left and right nilpotent, according to results in [5].

Problem 5. If $\mathbf{A}$ is a finite algebra, must there exist an integer n such that if θ is a minimal congruence in an algebra of $\mathcal{V}(\mathbf{A})$, and if θ is Abelian but not strongly Abelian, then every θ-equivalence class has at most n elements?

Status. Solved affirmatively.

Notes. K. A. Kearnes, E. W. Kiss, Á. Szendrei and R. Willard showed in [8] that $n = |A|^{|A|}$ works. (In fact, they obtained a smaller bound expressible in terms of the chief factor size of $\mathbf{A}$.)

Problem 6. Suppose that $\mathbf{L}$ is a finitely projective finite lattice. Does there exist a (locally finite) variety $\mathcal{W}$ with the property that for every locally finite variety $\mathcal{V}$ we have: $\mathcal{V}$ satisfies some idempotent Maltsev condition not satisfied by $\mathcal{W}$ iff $\mathbf{L}$ is not isomorphic to a sublattice of the congruence lattice of any algebra in $\mathcal{V}$?

Status. Some information is known.

Notes. The answer may depend on the lattice $\mathbf{L}$. Theorem 9.6(2)$\Leftrightarrow$(4) of [3] proves that if $\mathbf{L} = \mathbf{D}_1$, then one can take $\mathcal{W}$ equal to the variety of sets. The proof given depends in an essential way on the assumption that only locally finite $\mathcal{V}$ are considered, but in a forthcoming monograph K. A. Kearnes and E. W. Kiss prove that Problem 6 has an affirmative answer with $\mathbf{L} = \mathbf{D}_1$ and $\mathcal{W}$ equal to the variety of sets even if one does not restrict to locally finite $\mathcal{V}$.

It is proved in [7] that if $\mathbf{L} = \mathbf{D}_2$, then one can take $\mathcal{W}$ to be the variety of semilattices. Again, the proof depends in an essential way on the assumption that $\mathcal{V}$ is locally finite, but that restriction is removed in the Kearnes–Kiss monograph mentioned in the previous paragraph.

For the lattice $\mathbf{L} = \mathbf{M}_3$ it is not known whether Problem 6 has an affirmative answer when $\mathcal{V}$ is restricted to be locally finite, but it clearly has a negative answer if $\mathcal{V}$ is not assumed to be locally finite. To see why this is so, let $\mathcal{V}_1$ be the variety of vector spaces over the 2-element field, let $\mathcal{V}_2$ be the variety of vector spaces over the 3-element field, and let $\mathcal{V}_3$ be the variety of algebras with a single ternary majority operation. The $\mathcal{V}_i$ satisfy the following idempotent Maltsev conditions respectively: there exist a ternary operation $t_i(x, y, z)$ for which

(1) $x = t_1(x, y, y) = t_1(y, y, x) = t_1(y, x, y)$. (Take $t_1(x, y, z) = x - y + z$.)
(2) $x = t_2(x, y, y) = t_2(y, y, x) = t_2(t_2(x, y, x), y, t_2(x, y, x))$. (Take $t_2(x, y, z) = x - y + z$.)
(3) $x = t_3(x, x, y) = t_3(x, y, x) = t(y, x, x)$. (Take $t_3(x, y, z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$.)

If there were a variety $\mathcal{W}$ associated to $\mathbf{L} = \mathbf{M}_3$, as in the problem statement, then since $\mathbf{M}_3$ is isomorphic to a sublattice of the congruence lattice of a vector space over a field of characteristic 2 it follows that $\mathcal{W}$ satisfies every idempotent Maltsev condition satisfied by $\mathcal{V}_1$. In particular, $\mathcal{W}$ satisfies the idempotent Maltsev condition (1) from above. Similarly, using vector spaces over a field of characteristic 3, we get that $\mathcal{W}$ satisfies the idempotent Maltsev condition (2) from above. But any variety satisfying the idempotent Maltsev conditions (1) and (2) also satisfies (3). (Nonconstructive proof: If $\mathcal{W}$ satisfies (1), then it is congruence permutable, hence congruence modular. On a block of any Abelian congruence of an algebra in $\mathcal{V}$ the operations t_1 and t_2 both act as the unique affine Abelian group operation, hence $t_1(x, y, z) = t_2(x, y, z)$ if x, y and z are from the same block of an Abelian congruence. The conflict in characteristics implies that Abelian congruences are trivial for any algebra in $\mathcal{W}$. By modular commutator theory it must be that $\mathcal{W}$ is congruence distributive. But any congruence permutable, congruence distributive variety has a majority operation, so $\mathcal{W}$ satisfies the idempotent Maltsev condition (3).) (Constructive proof: take $t_3(x, y, z) = t_2(t_2(x, t_1(x, y, z), z), y, t_2(x, t_1(x, y, z), z))$. It works.) Now if $\mathcal{W}$ satisfied the conditions of the problem for $\mathbf{L} = \mathbf{M}_3$, then since $\mathcal{W}$ satisfies every idempotent Maltsev condition true in $\mathcal{V}_3$ we conclude that $\mathcal{V}_3$ has an algebra with an $\mathbf{M}_3$ embedded in its congruence lattice. But since algebras with a majority operation are congruence distributive, no algebra with a majority operation can have an $\mathbf{M}_3$ embedded in its congruence lattice. This contradiction shows that Problem 6 has a negative answer for $\mathbf{L} = \mathbf{M}_3$ if nonlocally finite $\mathcal{V}$ are permitted.

If we stick to the wording of the problem, this "counterexample" is not allowed because $\mathcal{V}_3$ is not locally finite. Indeed, is seems probable that their is a variety $\mathcal{W}$ associated to $\mathbf{M}_3$ for locally finite $\mathcal{V}$. We conjecture that one can take for $\mathcal{W}$ the join in the lattice of interpretability types of the varieties of affine modules over all rings of the form $M_n(\mathbb{Z}_p)$. We conjecture that if a *locally finite* variety $\mathcal{V}$ is interpretable into this join, then it is interpretable into a single joinand. If this is so, then this $\mathcal{W}$ works for $\mathbf{L} = \mathbf{M}_3$.

Problem 7. Let $\mathbf{L}$ be the congruence lattice of a finite algebra. Find interesting conditions under which the maximal intervals in $\mathbf{L}$ that omit type $\mathbf{4}$ are the equivalence classes of a congruence on $\mathbf{L}$. (Consider, for instance, these conditions: (1) The algebra belongs to a variety that omits types $\mathbf{1}$ and $\mathbf{5}$. (2) The algebra belongs to a variety in which type $\mathbf{4}$ minimal sets have empty tails.)

Status. Solved?

Notes. The construction in [13]******* produces a variety of type-set $\{\mathbf{3}, \mathbf{4}\}$ witnessing that condition (1) is insufficient to imply that maximal intervals in $\mathbf{L}$ that omit type $\mathbf{4}$ are the equivalence classes of a congruence on $\mathbf{L}$. One the other hand, condition (2) is easily seen to be sufficient to imply that maximal intervals in $\mathbf{L}$ that omit type $\mathbf{4}$ are the equivalence classes of a congruence on $\mathbf{L}$. (If (2) holds, then the intersection of the kernels of all restriction maps to type $\mathbf{4}$ minimal sets is a congruence on $\mathbf{L}$ whose equivalence classes are the maximal intervals that omit type $\mathbf{4}$.)

It seems plausible that this question was raised to encourage work on the congruence identity problem. See the notes on Problems 13 and 14.

Problem 8. Investigate minimal tolerances in finite algebras.

Status. Unsolved.

Notes. There exist dozens of papers in the literature that concern tolerances on finite algebras, but to my knowledge none of these papers constitute an attack on this problem.

Problem 9. Apply tame congruence theory to the study of minimal varieties.

Status. Partially solved.

Notes. See the notes on the next problem.

Problem 10. Describe all finite simple algebras that generate minimal varieties and possess no nontrivial subalgebras.

Status. Partially solved.

Notes. The Abelian case was settled by K. A. Kearnes, E. W. Kiss and M. A. Valeriote in [9], and independently by Á. Szendrei in [23, 24]. The description in this case is very explicit. We have a complete list of Abelian algebras (up to term equivalence) that satisfy the stated conditions.

The nonabelian case is dealt with by Kearnes and Szendrei in [10]. Their description has the following form: If $\mathbf{A}$ is finite, simple, nonabelian, and has no proper nonotrivial subalgebras, then $\mathbf{A}$ generates a minimal variety if and only if it satisfies a certain Maltsev-type condition. (Because there is room to sharpen this result, we report the status of this problem as "partially solved".)

Problem 11. Prove or disprove: If $\mathbf{A}$ is a finite algebra such that $\mathcal{V}(\mathbf{A})$ admits no finite bound for the cardinals of its simple algebras, this class of cardinals is not bounded by any cardinal.

Status. Solved negatively.

Notes. In [25], M. A. Valeriote produced an 8-element counterexample by modifying the algebra McKenzie constructed to solve Problem 12.

Problem 12. The same problem as 11, for subdirectly irreducible algebras in place of simple algebras.

Status. Solved negatively.

Notes. In [20], R. McKenzie exhibits a 4-element algebra $\mathbf{A}_\omega$ of infinite type that generates a residually finite variety with no finite bound for the cardinals of its subdirectly irreducible algebras. He also exhibits an 8-element algebra $\mathbf{C}$ with seven basic operations that generates a residually countable variety that is not residually finite.

Problem 13. Does there exist a locally finite variety that omits types $\mathbf{1}$ and $\mathbf{5}$ and whose class of congruence lattices obeys no nontrivial lattice equation?

Status. Solved negatively.

Notes. It is proved in Theorem 9.18 of [3] that if a locally finite variety obeys a nontrivial congruence identity, then it omites types $\mathbf{1}$ and $\mathbf{5}$. In Theorem 9.19 of [3] it is proved that if a locally finite variety omits types $\mathbf{1}, \mathbf{4}$ and $\mathbf{5}$, then it obeys a nontrivial congruence identity. Problems 13 and 14 concern the role of type $\mathbf{4}$ in the characterization of locally finite varieties that satisfy a nontrivial congruence identity. K. A. Kearnes proved in [6] that a locally finite variety $\mathcal{V}$ obeys a nontrivial congruence identity if and only if $\mathrm{typ}\{\mathcal{V}\} \cap \{\mathbf{1}, \mathbf{5}\} = \emptyset$.

Problem 14. Prove or disprove: If $\mathcal{V}$ is a locally finite variety, the class of congruences lattices of algebras in $\mathcal{V}$ obeys some nontrivial lattice equation iff $\mathcal{V}$ omits types $\mathbf{1}$ and $\mathbf{5}$, and also the type $\mathbf{4}$ minimal sets have empty tails.

Status. Solved negatively.

Notes. K. A. Kearnes and M. A. Valeriote constructed a counterexample in [13] by slightly modifying Polin's variety. Their example is a variety of type-set $\{\mathbf{3}, \mathbf{4}\}$ that satisfies all congruence identities true in Polin's variety, but some type $\mathbf{4}$ minimal sets have tails.

Problem 15. Investigate $\langle 0, \alpha\rangle$-minimal sets for Abelian minimal congruences α of finite groups. Do the same for rings.

Status. Unsolved for groups. Solved for rings.

Notes. There are a lot of unexplored questions concerning the interpretation of tame congruence theoretic concepts in classical settings. Some work has been done on describing minimal sets in classical structures, and further work has been done on the more restricted problem of determining which algebras in a given variety are E-minimal.

It follows from Theorem 13.9 of [3] that an E-minimal group must be a p-group (the type must be **2**, so Theorem 13.9 shows that it has prime power order). Conversely, Exercise 4.37(6) of [3] proves that any finite p-group is E-minimal. Thus, [3] already contains a classification of E-minimal groups. The classification of E-minimal semigroups is more complicated. S. W. Seif proved in [22] that a finite semigroup is E-minimal if and only if it is a p-group, a left or right zero semigroup, a nilpotent semigroup, or a 2-element semilattice. An unpublished result of P. Johnson and S. W. Seif is that a finite loop is E-minimal if and only if it is nilpotent and has prime power order.

K. A. Kearnes, E. W. Kiss and Cs. Szabó proved in 1996 that if $\mathbf{G}$ is a finite group, then any p-Sylow subgroup of $\mathbf{G}$ is the image of an idempotent polynomial. Moreover, if $\alpha \prec \beta$ are congruences on $\mathbf{G}$, then $\alpha|_P \neq \beta|_P$ for some Sylow subgroup P. Thus, any minimal set of $\mathbf{G}$ is polynomially isomorphic to one contained inside a Sylow subgroup. If $\mathbf{G}$ is nilpotent, then it is easy to see from this (and from remarks in the previous paragraph) that the minimal sets of $\mathbf{G}$ are exactly the cosets of Sylow subgroups. When $\mathbf{G}$ is not nilpotent, then determination of minimal sets has not been accomplished. If $\langle \alpha, \beta \rangle$ is Abelian, then the $\langle \alpha, \beta \rangle$-minimal sets have p-power order for some prime p, and at least one is contained in a p-Sylow subgroup for the same p, so it may be that each Abelian quotient has a minimal set that is a p-subgroup.

The classification of E-minimal modules follows from Exercise 13.10(3) of [3]. The determination of the $\langle \alpha, \beta \rangle$-minimal sets of finite modules, bimodules and rings can be found in the paper [2] by L. Conaway and K. A. Kearnes.

Problem 16. Let $\mathbf{A}$ be an E-minimal algebra with congruence lattice L. Is it true that $\mathbf{L}$ obeys every lattice equation that holds in all subgroup lattices of finite Abelian groups?

Status. Solved negatively.

Notes. P. P. Pálfy and Cs. Szabó proved that there is a congruence identity that holds in the variety of Abelian groups that fails to hold in the congruence lattice of any free group of more than 5 generators in the variety generated by the quaternion group. (Any p-group is E-minimal.) Problem 16 remains open if one modifies the second sentence to read "Is it true that $\mathbf{L}$ obeys every lattice equation that holds in all normal subgroup lattices of finite groups?"

Problem 17. Explain why it happens that Maltsev properties involving congruences almost invariably are expressible with operations of just three variables.

Status. Unsolved.

Notes. No one seems to have worked on this problem.

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